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# Anomalous couplings of vector bosons and the decay $H \longrightarrow \gamma\gamma$: Dimensional regularization versus momentum cutoff

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**Abstract**

As an illustration of general principles, the $W$-boson loop contribution to the amplitude for the decay $H \to \gamma\gamma$ is calculated within a specific model for the effective lagrangian describing the anomalous gauge boson couplings. Different approaches to the dependence of the result on the scale of new physics $\Lambda$ are briefly discussed.
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## 1 Introduction

It is widely believed that the standard model (SM) of electroweak interactions [1], although so far in beautiful agreement with experimental data, is not a final theory, but rather an effective low energy theory[1], valid for energies well below some scale $\Lambda$, which characterizes the onset of a new physics. This scale could be e.g. the mass of yet unobserved heavy particles present in the spectrum of the more fundamental theory. Integrating out these high energy degrees of freedom, one ends up with an effective lagrangian which describes only the interactions of the particles belonging to the low energy part of the original spectrum. Such an effective lagrangian should reflect the most general features of the original theory such as various types of symmetries, modes of their realization, anomaly matching conditions etc. On the other hand it is not constrained to be renormalizable in the usual sense, i.e. it can (and in general it has to, in order to be consistent) contain interaction terms with canonical dimension larger then four. These are multiplied with coupling constants, proportional to the inverse powers of some mass parameter related to the scale $\Lambda$, and therefore suppressed with respect to the renormalizable terms (the SM should be understood within this framework as the lowest order part of the complete effective lagrangian within the expansion in the powers of $\Lambda^{-1}$ ).The low energy coupling constants (LEC) could be at least in principle calculable (e.g. by integrating the heavy degrees of freedom order by order in perturbation theory and imposing necessary matching conditions at the threshold of new physics) provided the fundamental theory were known. However in the most of applications of the effective lagrangian approach this is not the case either because the explicit calculations are not possible, or, as in the case of SM, the fundamental theory of the new physics is not known. Rather one lets these parameters a priori undetermined and treat them as a useful parameterization of the dynamics of the yet unknown fundamental theory. Relaxing the constraint of the renormalizability of the interactions does not mean at all the complete loss of predictivity, which was the traditional argument for rejecting such theories. As a rule, within the framework of the effective theories there are well defined expansion prescriptions, which enable one to calculate loops and to absorb the infinities in the finite number of renormalized LEC at each order. These may be in principle measured experimentally and then used as an input for other predictions. Going to higher orders in the expansion, the number of LEC increases considerably, however their importance decreases because of suppression by negative powers of $\Lambda$. One can adopt also another point of view and use the measurement of various LEC as the experimental tests of variants of the models of the new physics.

There are generally two different types of effective lagrangians parametrizing the physics beyond the SM, corresponding to two different scenarios of breaking the gauge symmetry $SU(2)_L \otimes U(1)_Y \to U(1)_{em}$. The "decoupling scenario" assumes that the scale of the new physics is much larger than the electroweak symmetry breaking scale, $\Lambda \gg v$ , (here $v$ is the vacuum expectation value of the Higgs doublet), the $SU(2)_L \otimes U(1)_Y$ symmetry is then linearly realized and the low energy spectrum is identical with that of the SM, including the Higgs particle, which is supposed to be relatively light. The new nonrenormalizable interactions are

[1]For a recent review and complete list of references on the application of the effective lagrangian approach within the theory of the electroweak interaction see e.g. [2] and references therein. For the general principles of the effective lagrangians see also [3].

organized according to the increasing canonical dimension:

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_i \frac{f_i^{(6)}}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_i \frac{f_i^{(8)}}{\Lambda^4}\mathcal{O}_i^{(8)} + \ldots \tag{1}$$

and includes all operators of dimension 6, 8, ... invariant with respect to the $SU(2)_L \otimes U(1)_Y$.

The "nondecoupling scenario" on the other hand corresponds to the case $\Lambda \approx 4\pi v$. The new physics is related to the symmetry breaking sector of the SM, the gauge symmetry is realized nonlinearly and the effective lagrangian reflects the dynamics of the would-be Goldstone bosons eaten by the gauge bosons (and via the equivalence theorem [4], it is related to the dynamics of the longitudinal components of $W^\pm$ and $Z$). The most economic form of such an effective lagrangian for the symmetry breaking sector is given in the form of the (gauged) nonlinear $\sigma$ model organized as a derivative expansion

$$\mathcal{L}_{eff}^{SB} = \frac{v^2}{4}\mathrm{tr}(D_\mu U^+ D^\mu U) + \ldots, \tag{2}$$

(where $U = \exp(\mathrm{i}\xi^a\tau^a/v) \in SU(2)$, $\tau^a$ are Pauli matrices and $\xi^a$ are the would-be Goldstone boson fields, $D_\mu U = \partial_\mu U - g\widehat{W}_\mu U - (\mathrm{i}g'/2)B_\mu U$ and $\widehat{W}_\mu = (1/2\mathrm{i})W_\mu^a\tau^a$); the ellipses here mean terms with four and more derivatives and/or gauge fields, invariant with respect to the local $SU(2)_L \otimes U(1)_Y$ gauge transformation. This has for the $U$ field the following form

$$U \to \exp(\mathrm{i}\alpha_L^a\frac{\tau^a}{2})U\exp(-\mathrm{i}\alpha_Y\frac{\tau^3}{2}).$$

In order to preserve the $\rho$ parameter to be close to one, the additional (global) symmetries are often imposed. The example of such a symmetry is the custodial $SU(2)_c$ symmetry, which is introduced as the unbroken subgroup of the symmetry breaking pattern $SU(2)_L \otimes SU(2)_R \to SU(2)_c$, completely analogous to the pattern of chiral symmetry breaking of QCD with two light quarks [6]. The field $U$ transforms under $SU(2)_L \otimes SU(2)_R$ according to

$$U \to \exp(\mathrm{i}\alpha_L^a\frac{\tau^a}{2})U\exp(-\mathrm{i}\alpha_R^b\frac{\tau^b}{2}).$$

and the custodial symmetry corresponds to the diagonal subgroup $\alpha_L^a = -\alpha_R^a$. Of course, gauging then the $SU(2)_L \otimes U(1)_Y$ subgroup means the explicit breaking of the $SU(2)_c$ by the terms which vanish for $g^{'} \to 0$. Note also, that there is no Higgs field included in this type of effective lagrangian. However, also in this case it is possible to extend the model to account for such a type of particle [5] adding to the lagrangian additional terms containing $SU(2)_L \otimes U(1)_Y$ invariant field $H$:

$$\begin{aligned}\mathcal{L}_{eff}^{H} &= \frac{1}{2}\partial_\mu H\partial^\mu H - V(H) + a\left(\frac{H}{v}\right)\frac{v^2}{4}\mathrm{tr}(D_\mu U^+ D^\mu U) \\ &\quad + b\left(\frac{H}{v}\right)^2\frac{v^2}{4}\mathrm{tr}(D_\mu U^+ D^\mu U) + \ldots,\end{aligned} \tag{3}$$

the ellipses stand for terms of higher order as well as for the interaction terms of $H$ field with SM fermions. The SM Higgs is recovered for

$$\begin{aligned}V(H) &= \frac{m_H^2}{8v^2}((H+v)^2 - v^2)^2 \\ a &= 2,\ b = 1.\end{aligned} \tag{4}$$

In the unitary gauge, both these scenarios lead to the $U(1)_{em}$ invariant effective lagrangian with anomalous couplings. E.g. in the gauge boson sector, there are besides other contributions the following triple gauge boson couplings, which are usually written in the form of phenomenological lagrangian (we have omitted possible C or CP violating terms)[2]

$$\mathcal{L}_{eff}^{WWV} = \sum_{V=\gamma,Z} g_V[\mathrm{i}g_1^V(W_{\mu\nu}^+W^\mu V^\nu - W_{\mu\nu}W^{+\mu}V^\nu) + \mathrm{i}\kappa_V W_\mu^+ W_\nu V^{\mu\nu} \tag{5}$$

$$+\mathrm{i}\frac{\lambda_V}{m_W^2}W_{\nu\mu}^+ W_\lambda^\mu V^{\lambda\nu}], \tag{6}$$

where $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$and analogously for $V$, $g_\gamma = -e$ and $g_Z = -g\cos\theta_W$. Note, that within the SM, $g_1^V = \kappa_V = 1$, $\lambda_V = 0$. The constants $\kappa_\gamma$ and $\lambda_\gamma$ can be interpreted in terms of the anomalous magnetic and quadrupole moment of the $W$ bosons, $g_1^V$ corresponds to the gauge $U(1)$ charges of the $W$ in the the units of $g_\gamma$ an $g_Z$. Recent constraints on these couplings [7] come from the studies of $W\gamma$ events at Fermilab Tevatron; the CDF and D0 results are $-1.6 < \Delta\kappa_\gamma < 1.8$ and $-0.6 < \lambda_\gamma < 0.6$. Direct measurement will be also available at LEP2 [8].

# 2 Divergences within the effective theory and the dependence on the scale of the new physics

The $U(1)_{em}$ invariant phenomenological lagrangians reviewed in the previous section were often used in various calculations and treated as specific models of the deviations from the SM physics. There was certain controversy in the literature concerning the treatment of the divergences, which appear in the loops with anomalous vertices. Some of the authors used in one or another way the momentum cutoff at the scale of new physics $\Lambda_{cutoff} = \Lambda_{NP} = \Lambda$. Within this approach, which seems to be very physically illuminating, the principle [9] was used, that the divergent graph cut off at the scale where the effective theory looses its validity due the onset of the new physics gives a lower bound to the actual value of the graph in the full theory. I.e., in practise, only the most divergent contribution of the loop integral to the given amplitude was kept and used as the estimate of the dependence of the full theory amplitude on the scale of the new physics.The appearance of the divergences was therefore interpreted as an indication, that the process under consideration is strongly sensitive to the scale of the new physics.

This approach has been criticized (see e.g. the paper [9]) because it is in conflict with the decoupling theorems and gives ambiguous results. Moreover, as usual, it highly overestimates the dependence of the result on the scale[3] $\Lambda$ .

However, there are explicit examples discussed e.g. in [9] illustrating that the momentum cutoff could yield correct results[4]. As usual, it is not possible to resolve whether this is the case before performing explicit calculations.

[2]In fact, in such a phenomenological lagrangian the coupling constant should be understood in general as $q$ dependent formfactors.

[3]$\Lambda$ appears in the results calculated within this approach in a positive power (or logarithm) coming from the power counting order of the divergent loops.

[4]The result depends strongly on the choice of the field variables. The correct dependence on the scale of the new physics can be read off from the cutoff dependence of the loops only using the "good" variables. However, there is no clear criteria how to decide, whether the choice of variables is "good" or "bad".

In this paper we would like to briefly illustrate this general situation by using an explicit example of the $W$-boson loops contribution to the process $H \to \gamma\gamma$ calculated within simple $U(1)_{em}$ invariant phenomenological lagrangian in the unitary gauge using two regularization schemes. We will show, that there are differences between the momentum cutoff approach and another approach based on dimensional regularization with minimal subtractions, which was advocated in [9] and which should be accepted as the procedure giving the correct answer. We will therefore conclude, that the process $H \to \gamma\gamma$ is not reckoned among the cases, which could be treated correctly within the momentum cutoff prescription in the unitary gauge.

The paper is organized as follows. In Section 2 we introduce a specific model of the effective lagrangian with anomalous gauge boson couplings and present the results for the amplitude $H \to \gamma\gamma$ calculated within the two above mentioned schemes. In Section 3 we discuss the result of the calculations from the general point of view which was sketched in the Introduction. The comparison of the two approaches with respect to the dependence of the decay amplitude on the scale of the new physics and the conclusions are presented in Section 4. The details of the calculation of the amplitude are postponed to the Appendix.

# 3 $H \to \gamma\gamma$ within the effective lagrangian approach - a specific model

The theoretical concentration on the decay $H \to \gamma\gamma$ in the recent literature [10] – [14] is motivated by the fact that this rare decay mode could serve as the main source of the experimental signal for the Higgs particle with the mass within the lower part of the intermediate mass range $m_Z < m_H < 2m_W$ on hadron colliders (e.g. LHC).

Within the Standard Model, this decay channel of the Higgs boson is described at lowest order by a sum of one-loop Feynman diagrams, this sum is ultraviolet finite (the reason is that there is no tree-level $H\gamma\gamma$ interaction in the SM lagrangian; SM is a renormalizable theory, so that the $H\gamma\gamma$ counterterms cannot be present). As a result, one gets for the decay amplitude an expression of the following form [10], the tensor structure of which reflects the Lorentz invariance and $U(1)_{em}$ gauge invariance:

$$\mathcal{M}(H \to \gamma\gamma) = \frac{e^2 g}{16\pi^2}\frac{1}{m_W}\,\varepsilon^\star_\mu(k)\,\varepsilon^\star_\nu(l)\left[\frac{1}{2}m_H^2 g^{\mu\nu} - k^\nu l^\mu\right]\sum_{i=s,f,g} N_{c_i} e_i^2 F_i. \tag{7}$$

The decay rate is then

$$\Gamma(H \to \gamma\gamma) = \frac{\alpha^3}{128\pi^2\sin^2\theta_W}\frac{m_H^3}{m_W^2}\left|\sum_{i=s,f,g} N_{c_i} e_i^2 F_i\right|^2. \tag{8}$$

Here

$$\begin{aligned} F_s &= \tau_s\left(1 - \tau_s I^2\right) \\ F_f &= -2\tau_f\left[1 + (1-\tau_f)\, I^2\right] \\ F_g &= 2 + 3\tau_g + 3\tau_g\,(2-\tau_g)\, I^2 \end{aligned} \tag{9}$$

are the contributions of the scalar, fermion and vector boson loops resp.,

$$\begin{aligned}
\tau_i &= 4\left(\frac{m_i}{m_H}\right)^2 \\
N_{c_i} &= 1 \ \text{ for } i = \text{leptons, scalars and vector bosons} \\
N_{c_i} &= 3 \ \text{ for } i = \text{quarks,}
\end{aligned} \tag{10}$$

$e_i$ is electric charge of the loop particle in units of $e$, and

$$I = \begin{cases} \arctan \frac{1}{\sqrt{\tau - 1}} & , \tau > 1 \\ \frac{1}{2}\left[\pi + i \ln\left|\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}\right|\right] & , \tau < 1 \end{cases}. \tag{11}$$

It was believed that this process could be significantly influenced by possible deviations from the Standard Model, mainly in the gauge boson sector. To parametrize the physics beyond the Standard Model one can employ the effective lagrangian approach sketched above. This was done in several papers (see e.g. [11] – [14]); the specific forms of the used effective lagrangians vary among different authors. Since the additional effective couplings are generally of a non-renormalizable type, the resulting decay amplitude is, in contrast to the SM, UV divergent, so that the loop integrals have to be regularized to obtain reasonable predictions. This is another point, at which the above mentioned papers differ.

In order to illustrate this general situation we would like to present here a complementary alternative to the treatment contained in the work [11], in which the $W$-boson contribution to the decay width $H \to \gamma\gamma$ was calculated within the effective lagrangian approach. Using the same form of an effective lagrangian as in [11], we would like to demonstrate the differences resulting from a different cutoff prescription. We will also shortly comment on the interpretation of the output of the explicit calculations.

Let us first briefly review the main results of [11] and the way they were obtained. As the phenomenological lagrangian describing the gauge boson sector it was used the $U(1)_{em}$ invariant lagrangian $\mathcal{L}$ introduced originally in [15] (cf. also [16] ), conserving $C$ and $P$ separately. In the same notation as in[11], this phenomenological lagrangian is given in the unitary gauge as a sum of three terms

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3, \tag{12}$$

where

$$\mathcal{L}_1 = -\frac{1}{2}\widehat{G}^\dagger_{\mu\nu}\widehat{G}^{\mu\nu} + m_W^2 W^\dagger_\mu W^\mu + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \tag{13}$$

$$\mathcal{L}_2 = -\frac{1}{4}\sum_{V=\gamma Z} \widehat{F}^{(V)}_{\mu\nu}\widehat{F}^{(V)\mu\nu} \tag{14}$$

$$\mathcal{L}_3 = \sum_{V=\gamma Z} \frac{i g_V \lambda_V}{m_W^2}\widehat{F}^{(V)}_{\mu\nu}\widehat{G}^{\dagger\mu\rho}\widehat{G}_\rho{}^\nu. \tag{15}$$

Here

$$\begin{aligned}
\widehat{G}_{\mu\nu} &= \left(\partial_\mu - i g_\gamma A_\mu - i g_Z Z_\mu\right) W_\nu - \left(\partial_\nu - i g_\gamma A_\nu - i g_Z Z_\nu\right) W_\mu \\
\widehat{F}^{(V)}_{\mu\nu} &= F^{(V)}_{\mu\nu} + i g_V \kappa_V \left(W^\dagger_\mu W_\nu - W^\dagger_\nu W_\mu\right)
\end{aligned}$$

$$
\begin{aligned}
F_{\mu\nu}^{(V)} &= \partial_\mu V_\nu - \partial_\nu V_\mu \\
g_\gamma &= e \\
g_Z &= g\cos\theta_W .
\end{aligned}
\tag{16}
$$

Let us note, that the triple gauge boson couplings correspond to (6) with $g_1^\gamma = g_1^Z = 1$. The relation $g_1^\gamma = 1$ is quite natural, because it expresses the conservation of the electric charge of $W$ bosons, i.e. the $WW\gamma$ couplings derived from $\mathcal{L}$ are independent linear combination of the most general $U(1)_{em}$ terms conserving $C$ and $P$. This is, however, not true for the $WW\gamma\gamma$ couplings; here the possible interaction terms are not parametrized by independent LEC but multiplied by very specific combinations of $\kappa_\gamma$ and $\lambda_\gamma$. For $\kappa_V = 1$ and $\lambda_V = 0$, the lagrangian $\mathcal{L}$ reduces to the lagrangian of SM in the unitary gauge.

In addition to the above lagrangian of the gauge sector, the Higgs boson was included with the standard couplings to $W$ boson pair,

$$
\mathcal{L}_{WWH} = g m_W W_\mu^+ W^\mu H. \tag{17}
$$

We can think about the above $U(1)_{em}$ invariant phenomenological lagrangian as being produced by fixing the unitary gauge in the $SU(2)_L \otimes U(1)_Y$ invariant lagrangian within both decoupling and nondecoupling scenarios. Within the nondecoupling scenario, there are three operators of the order $\mathcal{O}(p^4)$ in the derivative expansion and three operators of the order $\mathcal{O}(p^6)$ needed to reproduce the structure of the lagrangian (12) in the unitary gauge, namely[5]

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{1}{2}\mathrm{tr}\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{v^2}{4}\mathrm{tr}(D_\mu U^+ D^\mu U) \\
& + \frac{1}{4g^2}(s_W^2\Delta\kappa_\gamma + c_W^2\Delta\kappa_Z)\mathrm{tr}(T[V_\mu, V_\nu])\mathrm{tr}(T[V^\mu, V^\nu]) \\
& - \frac{1}{2g}(s_W^2\Delta\kappa_\gamma + c_W^2\Delta\kappa_Z)\mathrm{tr}(T\widehat{W}_{\mu\nu})\mathrm{tr}(T[V^\mu, V^\nu]) \\
& + \frac{\mathrm{i}}{2g}s_W c_W(\Delta\kappa_\gamma - \Delta\kappa_Z)B_{\mu\nu}\mathrm{tr}(T[V^\mu, V^\nu]) \\
& + \frac{2}{3}\frac{g}{m_W^2}(s_W^2\lambda_\gamma + c_W^2\lambda_Z)\mathrm{tr}(\widehat{W}_{\nu\mu}\widehat{W}^{\mu\rho}\widehat{W}_\rho{}^\nu) \\
& - \mathrm{i}\frac{g}{m_W^2}s_W c_W(\lambda_\gamma - \lambda_Z)B_{\nu\mu}\mathrm{tr}(T\widehat{W}^{\mu\rho}\widehat{W}_\rho{}^\nu) \\
& - \frac{1}{m_W^2}(s_W\lambda_\gamma\Delta\kappa_\gamma + c_W\lambda_Z\Delta\kappa_Z)\mathrm{tr}(T[V_\nu, V_\mu])\mathrm{tr}(T\widehat{W}^{\mu\rho}\widehat{W}_\rho{}^\nu),
\end{aligned}
\tag{18}
$$

[5]In fact, the operator $\mathrm{tr}(T[V_\mu, V_\nu])\mathrm{tr}(T[V^\mu, V^\nu])$ can be expressed in terms of the following elements of the operator basis introduced in [20]:

$$
\begin{aligned}
\mathrm{tr}(T[V_\mu, V_\nu])\mathrm{tr}(T[V^\mu, V^\nu]) \;=\; & 4([\mathrm{tr}(V_\mu V_\nu)]^2 - [\mathrm{tr}(V_\mu V^\mu)]^2 - \mathrm{tr}(V_\mu V_\nu)\mathrm{tr}(TV^\mu)\mathrm{tr}(TV^\nu) \\
& + \mathrm{tr}(V_\mu V^\mu)\mathrm{tr}(TV_\nu)\mathrm{tr}(TV^\nu)).
\end{aligned}
$$

The same can be done with the operator $\mathrm{tr}(T[V_\nu, V_\mu])\mathrm{tr}(T\widehat{W}^{\mu\rho}\widehat{W}_\rho{}^\nu)$, here

$$
\begin{aligned}
\mathrm{tr}(T[V_\nu, V_\mu])\mathrm{tr}(T\widehat{W}^{\mu\rho}\widehat{W}_\rho{}^\nu) \;=\; & \mathrm{tr}(V_\nu V_\mu)\mathrm{tr}(\widehat{W}^{\mu\rho}\widehat{W}_\rho{}^\nu) - \mathrm{tr}(V_\mu\widehat{W}^{\mu\rho})\mathrm{tr}(\widehat{W}_\rho{}^\nu V_\nu) \\
& + 2\mathrm{tr}(TV_\mu)\mathrm{tr}(T\widehat{W}^{\mu\rho})\mathrm{tr}(\widehat{W}_\rho{}^\nu V_\nu) - 2\mathrm{tr}(T\widehat{W}_\rho{}^\nu)\mathrm{tr}(TV_\mu)\mathrm{tr}(V_\nu\widehat{W}^{\mu\rho})
\end{aligned}
$$

where

$$
\begin{aligned}
T &= U\tau^3U^+,\ V_\mu=(D_\mu U)U^+,\ D_\mu U=\partial_\mu U-g\widehat{W}_\mu U+g^{'}U\widehat{B}_\mu,\\
\widehat{W}_\mu &= \frac{1}{2\mathrm{i}}W^a_\mu\tau^a,\ \widehat{B}_\mu=\frac{1}{2\mathrm{i}}B_\mu\tau^3,\\
\widehat{W}_{\mu\nu} &= \partial_\mu\widehat{W}_\nu-\partial_\nu\widehat{W}_\mu-g[\widehat{W}_\mu,\widehat{W}_\nu], B_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu
\end{aligned}
$$

and $\Delta\kappa_V=\kappa_V-1$. The lagrangian (17) can be obtained in the same way from $\mathcal{O}(p^2)$ term

$$
\mathcal{L}_{WWH}=\frac{v^2}{2}\left(\frac{H}{v}\right)\mathrm{tr}(D_\mu U^+D^\mu U).
$$

Within the decoupling scenario, the same anomalous gauge boson couplings are reproduced by the lagrangian with two dimension 6, three dimension 8 and one dimension 10 operators (cf. e.g. [17]) :

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2}\mathrm{tr}\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}+D_\mu\Phi^+D^\mu\Phi\\
&+\frac{4\mathrm{i}}{gv^2}s_Wc_W(\Delta\kappa_\gamma-\Delta\kappa_Z)B_{\mu\nu}D^\mu\Phi^+D^\nu\Phi\\
&+\frac{2}{3}\frac{g}{m_W^2}(s_W^2\lambda_\gamma+c_W^2\lambda_Z)\mathrm{tr}(\widehat{W}_{\nu\mu}\widehat{W}^{\mu\rho}\widehat{W}_\rho{}^\nu)\\
&+\frac{16}{gv^4}(s_W^2\Delta\kappa_\gamma+c_W^2\Delta\kappa_Z)\Phi^+\widehat{W}_{\nu\mu}\Phi D^\mu\Phi^+D^\nu\Phi\\
&+\frac{4}{gv^4}(s_W^2\Delta\kappa_\gamma+c_W^2\Delta\kappa_Z)[D^\mu\Phi^+D^\nu\Phi-D^\nu\Phi^+D^\nu\Phi]^2\\
&+\frac{4\mathrm{i}}{m_W^2v^2}s_Wc_W(\lambda_\gamma-\lambda_Z)B_{\nu\mu}\Phi^+\widehat{W}^{\mu\rho}\widehat{W}_\rho{}^\nu\Phi\\
&+\frac{16}{m_W^2v^4}(s_W\lambda_\gamma\Delta\kappa_\gamma+c_W\lambda_Z\Delta\kappa_Z)\\
&\times(D^\mu\Phi^+D^\nu\Phi-D^\nu\Phi^+D^\nu\Phi)\Phi^+\widehat{W}^{\mu\rho}\widehat{W}_\rho{}^\nu\Phi, \qquad (19)
\end{aligned}
$$

Here $\Phi$ is the SM Higgs doublet , $s_W=\sin\theta_W$, $c_W=\cos\theta_W$ and $D^\nu\Phi=\partial_\mu\Phi-g\widehat{W}_\mu\Phi+\mathrm{i}g^{'}/2B_\mu\Phi$. The standard Higgs boson coupling (17) stems now from the third term of (19). Such a lagrangian produces, however, also anomalous Higgs boson couplings, which were not considered in [11].

Both the nondecoupling and decoupling interpretations of the origin of the phenomenological lagrangian (12) illustrate again the fact, that this lagrangian is incomplete in the sense, that it does not contain all the independent linear combinations of the full set of the operators up to a given dimension or number of (covariant) derivatives[6]. Nevertheless, we use it here as it stands for the illustrative purposes, mainly because of the relative calculational simplicity.

[6]Let us also note that the suppression of the higher dimension or higher order operators does not correspond to the usual factors of the type $1/\Lambda^2$ for the decoupling scenario and $1/(4\pi v)$ for the nondecoupling scenario, but rather to the factor $g^2/m_W^2\sim 1/v^2$ . The reason is, that the contributions of all the operators to the parameters $\kappa_\gamma$ and $\lambda_\gamma$ should have the same order of magnitude in order to reproduce the lagrangian 12.

In the paper [11], only the $WWH$, $WW\gamma$ and $WW\gamma\gamma$ vertices derived from the above lagrangian (12) were used in the calculation of the $W$ -boson contribution to the $H \to \gamma\gamma$ decay width. The calculation was performed in the unitary gauge. There are two types of Feynman diagrams, namely the triangle (and corresponding cross diagram) and the tadpole; both of them, when regulated using momentum cutoff, lead to the quadratic divergent loop integrals. The sum of these diagrams remain UV divergent unless $\kappa_\gamma = 1$ and $\lambda_\gamma = 0$, as it was explained above. The result given in [11] was presented in the form of the sum of the (cutoff independent) SM expression and the (cutoff dependent) correction. The latter was identified with the quadratic divergence of the diagrams with at least one anomalous vertex proportional $\kappa_\gamma - 1$ and/or $\lambda_\gamma$. I.e.

$$\begin{aligned} F_g &= 2 + 3\tau_g + 3\tau_g\,(2-\tau_g)\,I^2 \\ &\quad + \left(\frac{\Lambda}{m_W}\right)^2 \left(3\Delta\kappa_\gamma - 4\lambda_\gamma + \lambda_\gamma^2 + \frac{1}{2}\Delta\kappa_\gamma^2\right), \end{aligned} \tag{20}$$

and the cutoff of the loop momentum $\Lambda$ was interpreted as the scale where the new physics comes in. Because the divergences are local, within this approach the effect of the loops with anomalous gauge boson coupling is equivalent to some direct $H\gamma\gamma$ interaction vertex. It is not difficult to see, that it corresponds to the vertex of the type

$$\mathcal{L}_{H\gamma\gamma} = G_{eff} e^2 \left(\frac{H}{v}\right) F_{\mu\nu} F^{\mu\nu},$$

where the effective coupling constant reads

$$G_{eff} = \frac{1}{(4\pi)^2} \left(\frac{\Lambda}{m_W}\right)^2 \left(3\Delta\kappa_\gamma - 4\lambda_\gamma + \lambda_\gamma^2 + \frac{1}{2}\Delta\kappa_\gamma^2\right). \tag{21}$$

As an alternative to this, let us present here the result of our calculation (the details of the calculation can be found in the Appendix) of the same quantity $F_g$ within dimensional regularization and $\overline{MS}$ subtraction scheme. Such a treatment was advocated in [9]. The result can be split up to the finite and divergent parts in the following way

$$F_g = F_g^{fin} + F_g^{div}, \tag{22}$$

where the finite part is

$$\begin{aligned} F_g^{fin} &= 3 - {\kappa_\gamma}^2 + 3\,\tau_g + 2\,\lambda_\gamma + 6\,\kappa_\gamma\,\lambda_\gamma - 5\,{\lambda_\gamma}^2 \\ &\quad + \Big(-1 + 2\,\kappa_\gamma - {\kappa_\gamma}^2 + 2\,\lambda_\gamma - 2\,\kappa_\gamma\,\lambda_\gamma - {\lambda_\gamma}^2 \\ &\quad + \left(3 + 2\,\kappa_\gamma + {\kappa_\gamma}^2 - 2\,\lambda_\gamma - 2\,\kappa_\gamma\,\lambda_\gamma + {\lambda_\gamma}^2\right)\,\tau_g - 3\,{\tau_g}^2\Big)\,I^2 \\ &\quad + (-3 - 2\,\kappa_\gamma + 5\,{\kappa_\gamma}^2 - 2\,\lambda_\gamma + 2\,\kappa_\gamma\,\lambda_\gamma + \frac{25}{3}\,{\lambda_\gamma}^2)\frac{1}{\tau_g} \\ &\quad + \Big(8 - 4\,\kappa_\gamma - 4\,{\kappa_\gamma}^2 + 8\,\lambda_\gamma + 8\,\kappa_\gamma\,\lambda_\gamma - 14\,{\lambda_\gamma}^2 \\ &\quad + (-4 + 4\,{\kappa_\gamma}^2 + 8\,{\lambda_\gamma}^2)\frac{1}{\tau_g} \end{aligned}$$

$$
+ \left(-4 + 4\,\kappa_\gamma - 8\,\lambda_\gamma - 8\,\kappa_\gamma\,\lambda_\gamma + 6\,{\lambda_\gamma}^2\right)\,\tau_g\Big)\,\frac{I}{J}
$$

$$
+ \left(-8\,\lambda_\gamma - 4\,\kappa_\gamma\,\lambda_\gamma + 6\,{\lambda_\gamma}^2 + (2 - 2\,{\kappa_\gamma}^2 - 4\,{\lambda_\gamma}^2)\frac{1}{\tau_g}\right)\,\ln\left(\frac{{m_W}^2}{\mu^2}\right) \tag{23}
$$

and the divergent part is

$$
\begin{aligned}
F_g^{div} &= \left(-8\,\lambda_\gamma - 4\,\kappa_\gamma\,\lambda_\gamma + 6\,{\lambda_\gamma}^2 + (2 - 2\,{\kappa_\gamma}^2 - 4\,{\lambda_\gamma}^2)\frac{1}{\tau_g}\right) \\
&\times \left(\frac{2}{4-\mathrm{D}} - \gamma_E + \ln(4\,\pi)\right).
\end{aligned} \tag{24}
$$

In these formulae $\mathrm{D} = 4 - 2\epsilon$ and

$$
J = \begin{cases} \sqrt{\tau_g - 1}, & \tau_g > 1 \\ -i\sqrt{1-\tau_g}, & \tau_g < 1 \end{cases}. \tag{25}
$$

## 4 Discussion

Let us now briefly discuss how to interpret this result. As we have shown above, the lagrangian (12) is a mixture of terms stemming from operators with different dimensions (from 6 up to 10 in the framework of the decoupling scenario) and different orders in the momentum expansion (from $\mathcal{O}(p^2)$ up to $\mathcal{O}(p^6)$ in the framework of the nondecoupling scenario). As a consequence, the formulas (23), (24) do not respect the hierarchy of contributions originating from the hierarchy of the tower of effective operators, which is the cornerstone of the consistent treatment of the nonrenormalizable couplings within the effective lagrangian approach. Therefore, in order to extract the partial information about the relevant dependence on the scale of the new physics $\Lambda$, it is necessary to reorganize the resulting formulas and to keep only the terms, which are dominant within the two possible scenarios reviewed in the Sec. 1.

Within the framework of the decoupling scenario, we expect that we can safely neglect[7] the operators of dimension 8 and higher and keep only operators of dimension 6. The lagrangian (19) contains two such operators (we use here the notation of [19]), namely

$$
\begin{aligned}
\mathcal{O}_B &= \frac{\mathrm{i}g'}{2} B_{\mu\nu} D^\mu \Phi^+ D^\nu \Phi, \\
\mathcal{O}_{WWW} &= -\frac{g^3}{3!}\mathrm{tr}(\widehat{W}_{\nu\mu}\widehat{W}^{\mu\rho}\widehat{W}_\rho{}^\nu),
\end{aligned} \tag{26}
$$

so that we can use (23,24) to get information about the contribution of these two operators only.

[7]However,as it was shown in [18], in the case when the fundamental full theory is weakly coupled gauge theory, the operators of dimension 8 should also contribute significantly provided they can be generated at the tree level. In this case their contribution is comparable with that of the one loop generated dimension 6 operators, which are multiplied by additional factor $1/16\pi^2$. This factor can be of the same size like the suppression factor $v^2/\Lambda^2$ of the dimension 8 operators, provided the scale of new physics is in the range of a few TeV.

As far as the $WW\gamma$ and $WW\gamma\gamma$ couplings used for the above calculation of the decay amplitude are concerned, the presence of the term $(\alpha_B/\Lambda^2)\,\mathcal{O}_B$ in the lagrangian generates effectively a contribution to the phenomenological parameter $\Delta\kappa_\gamma$ (and not to the $\lambda_\gamma$)

$$\Delta\kappa_\gamma^B = \frac{1}{2}\frac{m_W^2}{\Lambda^2}\alpha_B, \tag{27}$$

while the term $(\alpha_{WWW}/\Lambda^2)\,\mathcal{O}_{WWW}$ contributes to the $\lambda_\gamma$ (and not to the $\Delta\kappa_\gamma$)

$$\lambda_\gamma^{WWW} = -\frac{1}{4}g^2\frac{m_W^2}{\Lambda^2}\alpha_{WWW}. \tag{28}$$

I.e., within the decoupling scenario, the natural values of the parameters $\Delta\kappa_\gamma$ and $\lambda_\gamma$ are of the order $\mathcal{O}(\frac{m_W^2}{\Lambda^2})$ and $\mathcal{O}(g^2\frac{m_W^2}{\Lambda^2})$ respectively[8] and the leading order anomalous contribution is therefore

$$\begin{aligned}
F_g^{fin} &= 2 + 3\tau_g + 3\tau_g\,(2-\tau_g)\,I^2 \\
&\quad -2\Delta\kappa_\gamma + 8\left(\lambda_\gamma + \Delta\kappa_\gamma\frac{1}{\tau_g}\right) - 4\lambda_\gamma\tau_g I^2 \\
&\quad +4\left[4\lambda_\gamma - 3\Delta\kappa_\gamma + 2\Delta\kappa_\gamma\frac{1}{\tau_g} + (\Delta\kappa_\gamma^B - 4\lambda_\gamma)\tau_g\right]\frac{I}{J} \\
&\quad -4(3\lambda_\gamma + \Delta\kappa_\gamma^B\frac{1}{\tau_g})\ln\left(\frac{{m_W}^2}{\mu^2}\right) + \mathcal{O}\left(\frac{1}{\Lambda^4}\right) \qquad (29) \\
F_g^{div} &= -4\left(3\lambda_\gamma + \Delta\kappa_\gamma\frac{1}{\tau_g}\right)\left(\frac{2}{4-\mathrm{D}} - \gamma_E + \ln(4\,\pi)\right). \qquad (30)
\end{aligned}$$

According to the renormalization prescription for the decoupling scenario, the divergent part should be cancelled by the contributions of the appropriate counterterms stemming from effective operators of dimension 6. In the unitary gauge, such a counterterm has the form[9]

$$\mathcal{L}_{H\gamma\gamma} = G_{H\gamma\gamma}\frac{v^2}{\Lambda^2}e^2\left(\frac{H}{v}\right)F_{\mu\nu}F^{\mu\nu}, \tag{31}$$

where $G_{H\gamma\gamma}$ is an effective (running) coupling constant[10]. This brings about the following

[8]Let us stress that within the full $SU(2)\times U(1)$ approach, the operator $\mathcal{O}_B$ would induce also the anomalous $HWW\gamma$ coupling; however within the lagrangian (19) this contribution is cancelled by the contributions of the higher dimension operators with unnaturally large coupling constants. That is, the lagrangian (19) does not allow to get the full information on the influence of the operator $\mathcal{O}_B$ on the process $H\to\gamma\gamma$ using the above formulas (23,24), which is restricted to the effect of the anomalous $WW\gamma$ and $WW\gamma\gamma$ vertices generated by $\mathcal{O}_B$ . On the other hand, the contribution of the loops with one insertion of the anomalous $WW\gamma$ and $WW\gamma\gamma$ vertices generated by dimension 6 operators (26) can be inferred therefore from (23,24) by means of the expansion to the order $\mathcal{O}(\Delta\kappa_\gamma, \lambda_\gamma)$.

[9]Here we explicitly factored out the suppression factor $v^2/\Lambda^2$ of the dimension 6 operators. There could be also additional suppression of the order $1/(4\pi)^2$ for the one loop generated dimension 6 operators.

[10]This form of interaction is generated e.g. by fixing the unitary gauge in the following dimension 6 operators

$$\mathcal{O}_{WW} \;=\; g^2\Phi^+\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\Phi,$$

additional contribution to the function $F_g$ :

$$F_g^{H\gamma\gamma} = \frac{(4\pi v)^2}{\Lambda^2} G_{H\gamma\gamma}. \tag{35}$$

.

The above result of the loop calculation can be also used to get information on the scale dependence of the effective constant $G_{H\gamma\gamma}$. In order to ensure the scale independent result for the decay rate it should hold[12]

$$G_{H\gamma\gamma}(\mu') = G_{H\gamma\gamma}(\mu) - \frac{\Lambda^2}{(4\pi v)^2} 8 \left(3\lambda_\gamma + \Delta\kappa_\gamma \frac{1}{\tau_g}\right) \ln\left(\frac{\mu'}{\mu}\right) + \dots$$

Within the framework of the nondecoupling scenario, the lowest order anomalous contribution can be obtained by keeping only the contribution of the $\mathcal{O}(p^4)$ operators. There are three such operators in the lagrangian (18). The remaining three operators are of order $\mathcal{O}(p^6)$, these operators are proportional to the parameters $\lambda_V$. I.e. setting $\lambda_\gamma \to 0$ in the formulas (23,24) and expanding to the first order in $\Delta\kappa_\gamma$ we get the leading order contribution to the decay amplitude from the very specific combination of the $\mathcal{O}(p^4)$ operators (here we use the same notation as in [20])

$$\begin{aligned}
\mathcal{L}_2 &= \mathrm{i}\frac{g'}{2} B_{\mu\nu}\mathrm{tr}(T[V^\mu, V^\nu]) \\
\mathcal{L}_4 &= [\mathrm{tr}(V_\mu V_\nu)]^2 \\
\mathcal{L}_5 &= [\mathrm{tr}(V_\mu V^\mu)]^2 \\
\mathcal{L}_6 &= \mathrm{tr}(V_\mu V_\nu)\mathrm{tr}(TV^\mu)\mathrm{tr}(TV^\nu) \\
\mathcal{L}_7 &= \mathrm{tr}(V_\mu V^\mu)\mathrm{tr}(TV_\nu)\mathrm{tr}(TV^\nu)) \\
\mathcal{L}_9 &= \mathrm{i}\frac{g}{2}\mathrm{tr}(T\widehat{W}_{\mu\nu})\mathrm{tr}(T[V^\mu, V^\nu]),
\end{aligned} \tag{36}$$

with coefficients given by (18), cf. also footnote 5. The trilinear vector boson coupling is generated by the operators $\mathcal{L}_2$ and $\mathcal{L}_9$. Explicitly, the presence of the terms

$$\mathcal{L} = \alpha_2 \frac{v^2}{\Lambda^2}\mathcal{L}_2 + \alpha_9 \frac{v^2}{\Lambda^2}\mathcal{L}_9 \tag{37}$$

$$\begin{aligned}
\mathcal{O}_{BB} &= -\frac{g'^2}{4}\Phi^+ B_{\mu\nu}B^{\mu\nu}\Phi, \\
\mathcal{O}_{BW} &= -\frac{\mathrm{i}gg'}{2}\Phi^+ B_{\mu\nu}\widehat{W}^{\mu\nu}\Phi.
\end{aligned} \tag{32}$$

Writing the corresponding terms of the effective lagrangian in the form

$$\mathcal{L}_{ct} = \frac{\alpha_{WW}}{\Lambda^2}\mathcal{O}_{WW} + \frac{\alpha_{BB}}{\Lambda^2}\mathcal{O}_{BB} + \frac{\alpha_{BW}}{\Lambda^2}\mathcal{O}_{BW} \tag{33}$$

we have[11]

$$G_{H\gamma\gamma} = -\frac{1}{4}(\alpha_{WW} + \alpha_{BB} - \alpha_{BW}), \tag{34}$$

Let us also note, that the operators (33) lead to the gauge boson wave function renormalization and mixing, as well as to the anomalous $HWW\gamma$ and $HWW\gamma\gamma$ vertices, which should also be included in the complete analysis of the process under consideration, however these effects are not discussed here.

[12] Within the full $SU(2) \times U(1)$ approach, the ellipses here would stand for the other terms coming from the other graphs with vertices generated by $\mathcal{O}_B$ as well as from other dimension 6 operators not considered here.

in the effective lagrangian give rise to the following contribution to the parameter $\Delta\kappa_\gamma$:

$$\Delta\kappa_\gamma = -g^2 \frac{v^2}{\Lambda^2}(\alpha_2 + \alpha_9) = -4\frac{m_W^2}{\Lambda^2}(\alpha_2 + \alpha_9), \tag{38}$$

i.e. the natural value for the coupling $\Delta\kappa_\gamma$ of the lagrangian (18) is of the order $\mathcal{O}(\frac{m_W^2}{\Lambda^2})$. For the leading order contribution to the function $F_g$ we get then

$$\begin{aligned} F_g^{fin} &= 2 + 3\tau_g + 3\tau_g\,(2-\tau_g)\,I^2 \\ &\quad +2\Delta\kappa_\gamma\Bigg( -1 + 4\frac{1}{\tau_g} + 2\left[-3 + 2\frac{1}{\tau_g} + \tau_g\right]\frac{I}{J} - 2\frac{1}{\tau_g}\ln\left(\frac{{m_W}^2}{\mu^2}\right)\Bigg) \\ &\quad +\mathcal{O}\left(\frac{1}{\Lambda^4}\right) \end{aligned} \tag{39}$$

$$F_g^{div} = -4\Delta\kappa_\gamma \frac{1}{\tau_g}\left(\frac{2}{4-\mathrm{D}} - \gamma_E + \ln(4\,\pi)\right). \tag{40}$$

Note, that the divergent part is proportional to the $m_H^2 = (k+l)^2$, this reflects the fact that in the nondecoupling case the divergencies should be canceled by $\mathcal{O}(p^6)$ counterterms. In the unitary gauge such a counterterm has the form[13]

$$\mathcal{L}_{H\gamma\gamma} \;=\; \widetilde{G}_{H\gamma\gamma}e^2\frac{v^2 m_H^2}{\Lambda^4}\left(\frac{H}{v}\right)F_{\mu\nu}F^{\mu\nu}, \tag{41}$$

where $\widetilde{G}_{H\gamma\gamma}$ is an effective coupling constant and we have

$$F_g^{H\gamma\gamma} = \frac{m_H^2}{\Lambda^2}\widetilde{G}_{H\gamma\gamma}. \tag{42}$$

Also in this case we can infer information on the running of this effective coupling; the scale dependence coming from the graphs with the above mentioned specific combination of $\mathcal{O}(p^4)$ operators should be[14]

$$\widetilde{G}_{H\gamma\gamma}(\mu') \;=\; \widetilde{G}_{H\gamma\gamma}(\mu) - 2\Delta\kappa_\gamma\left(\frac{\Lambda^2}{4\pi v m_W}\right)^2 \ln\left(\frac{\mu'}{\mu}\right) + \ldots$$

# 5 Conclusions

Let us now compare these results with those of ref. [11]. Inserting the dimensional analysis estimates (27, 28, 38) of the parameters $\Delta\kappa_\gamma$ and $\lambda_\gamma$ within both scenarios to the cutoff analysis formula and using the principle quoted in the Sec.2 (20), we get the following lower bound on the dependence of the function $\Delta F_g$ on the scale of the new physics:

[13] Here again the suppression factor $v^2 m_H^2/\Lambda^4$ corresponding to the naive dimensional analysis was factored out. The natural value of the symmetry breaking scale is $\Lambda \sim 4\pi v$.

[14] Here, within the full nondecoupling approach, the ellipses would mean the contributions of operators not listed above.

$$\Delta F_g^{\rm cutoff} = \frac{3}{2}\alpha_B + g^2\alpha_{WWW} + \mathcal{O}\left(\frac{m_W^2}{\Lambda^2}\right) \tag{43}$$

for the decoupling scenario and

$$\Delta F_g^{\rm cutoff} = -12(\alpha_2 + \alpha_9) + \mathcal{O}\left(\frac{m_W^2}{\Lambda^2}\right) \tag{44}$$

for the nondecoupling scenario. I.e. in this scheme the natural values of the anomalous contribution are (independently on the scale of the new physics) $\mathcal{O}(1)$ , provided the natural values of the LEC $\alpha_B$, $\alpha_{WWW}$, $\alpha_2$ and $\alpha_9$ are of the order $\mathcal{O}(1)$. On the other hand, within the dimensional regularization approach the dominant anomalous contribution to the function $F_g$ comes from the direct interaction terms (cf. formulas (35, 42)) and the $\mu$ dependent loop logarithms. For the purpose of the dimensional analysis, we can (using the equations for the running of the LEC) interpret the scale $\mu$ as a scale at which the constants $G_{H\gamma\gamma}$and $\widetilde{G}_{H\gamma\gamma}$ acquire their natural values of order $\mathcal{O}(1)$. This is expected to correspond to the point at which the underlying high energy theory is matched with the low energy effective lagrangian, i.e. to the scale of the new physics $\Lambda$. We have then the following estimate

$$\begin{aligned}
\Delta F_g^{\rm DR} &= \frac{(4\pi v)^2}{\Lambda^2}G_{H\gamma\gamma} - \left(\frac{1}{2}\frac{m_H^2}{\Lambda^2}\alpha_B - 3g^2\frac{m_W^2}{\Lambda^2}\alpha_{WWW}\right)\ln\left(\frac{{m_W}^2}{\Lambda^2}\right) + \ldots \\
&= \mathcal{O}\left(\frac{(4\pi v)^2}{\Lambda^2}\right) + \mathcal{O}\left(\frac{m_H^2}{\Lambda^2}\ln\left(\frac{{m_W}^2}{\Lambda^2}\right)\right) + \ldots
\end{aligned}$$

for the decoupling case (there could be an overall factor $1/(4\pi)^2$ for the loop generated dimension 6 operator contribution) and

$$\begin{aligned}
\Delta F_g^{\rm DR} &= \frac{m_H^2}{\Lambda^2}\widetilde{G}_{H\gamma\gamma} + 4(\alpha_2+\alpha_9)\frac{m_H^2}{\Lambda^2}\ln\left(\frac{{m_W}^2}{\Lambda^2}\right) + \ldots \\
&= \mathcal{O}\left(\frac{m_H^2}{\Lambda^2}\right) + \mathcal{O}\left(\frac{m_H^2}{\Lambda^2}\ln\left(\frac{{m_W}^2}{\Lambda^2}\right)\right) + \ldots
\end{aligned}$$

for the nondecoupling case. The ellipses here mean further terms, unimportant from the numerical point of view.

We can make the following conclusion.The above formulas show, that the cutoff scheme is in disagreement with dimensional analysis approach in this special case of the calculation of the process $H \to \gamma\gamma$. In the often considered cases (e.g. $\Lambda = 1TeV$ for the decoupling scenario and loop generated dimension 6 operators, or $\Lambda \leq 4\pi v$ and $m_H < \Lambda$ for the nondecoupling scenario) the formulas (43), (44) overestimate the enhancement or suppression of the decay rate of the process under consideration. Therefore, this explicit example does not rank among the cases, which could be treated correctly within the momentum cutoff prescription. This corresponds to the general expectations expressed in the ref. [9], where further examples of both the failure and success of cutoff analysis within the unitary gauge can be found.

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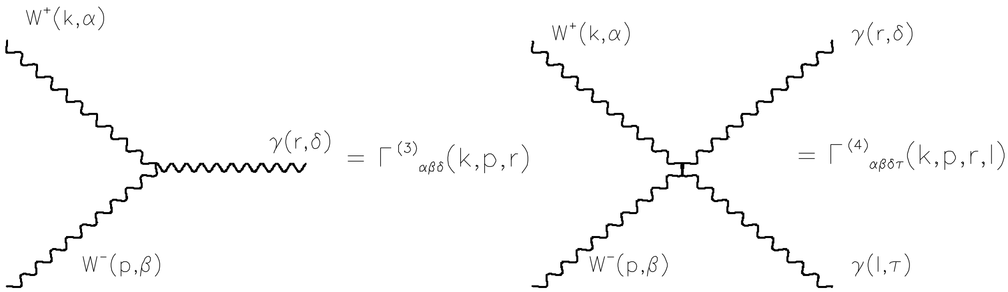


Figure 1: Feynman rules for the $WW\gamma$ and $WW\gamma\gamma$ couplings. All the momenta are in-going. The explicit expressions for the vertex functions are given in the Appendix.

## Appendix

In this Appendix, we display some explicit formulas illustrating the calculation of the decay rate $H \to \gamma\gamma$ within the framework of the dimensional regularization. The $W$ -boson propagator in the unitary gauge is given by the formula

$$\Delta_F^{\mu\nu}(k) = -\frac{\mathrm{i}\left(g^{\mu\nu} - \frac{k_\mu k_\nu}{m_W^2}\right)}{k^2 - m_W^2 + \mathrm{i}\varepsilon}$$

and the Feynman rules for the $HWW$, $WW\gamma$ and $WW\gamma\gamma$ couplings are depicted in Fig. 1 The corresponding vertex functions are

$$\begin{aligned}
\Gamma^{(3)}_{\alpha\beta\delta}(k,p,r) &= \mathrm{i}e\{g_{\alpha\beta}(p-k)_\delta + g_{\alpha\delta}(k-r)_\beta + g_{\beta\delta}(r-p)_\alpha \\
&\quad + \Delta\kappa_\gamma(g_{\beta\delta}r_\alpha - g_{\alpha\delta}r_\beta) \\
&\quad + \frac{\lambda_\gamma}{m_W^2}[(k\cdot p)(g_{\beta\delta}r_\alpha - g_{\alpha\delta}r_\beta) + (r\cdot k)(g_{\alpha\beta}p_\delta - g_{\beta\delta}p_\alpha) \\
&\quad + (r\cdot p)(g_{\alpha\delta}k_\beta - g_{\alpha\beta}k_\delta) + r_\beta k_\delta p_\alpha - r_\alpha k_\beta p_\delta]\} \qquad (45)
\end{aligned}$$

and

$$\begin{aligned}
\Gamma^{(4)}_{\alpha\beta\delta\tau}(k,p,r,l) &= -\mathrm{i}e^2(2g_{\alpha\beta}g_{\delta\tau} - g_{\alpha\delta}g_{\beta\tau} - g_{\alpha\tau}g_{b\delta}) \qquad (46) \\
&\quad + \frac{\mathrm{i}e^2\lambda_\gamma}{m_W^2}\{-g_{\alpha\beta}g_{\delta\tau}[(r+l)\cdot(k+p)] \\
&\quad + g_{\alpha\delta}g_{\beta\tau}[(r\cdot p) + (l\cdot k)] \\
&\quad + g_{\alpha\tau}g_{\beta\delta}[(r\cdot k) + (l\cdot p)] \\
&\quad + g_{\alpha\delta}[(k-p)_\tau r_\beta - r_\tau k_\beta - l_\beta k_\tau] \\
&\quad + g_{\alpha\tau}[(k-p)_\delta l_\beta - r_\beta p_\delta - l_\delta k_\beta] \qquad (47) \\
&\quad + g_{\beta\delta}[-(k-p)_\tau r_\alpha - r_\tau p_\alpha - l_\alpha p_\tau] \\
&\quad + g_{\beta\tau}[-(k-p)_\delta l_\alpha - l_\delta p_\alpha - r_\alpha p_\delta] \\
&\quad + g_{\alpha\beta}[(k+p)_\delta r_\tau + (k+p)_\tau l_\delta] \\
&\quad + g_{\delta\tau}[k_\beta(r+l)_\alpha + p_\alpha(r+l)_\beta]\}. \qquad (48)
\end{aligned}$$

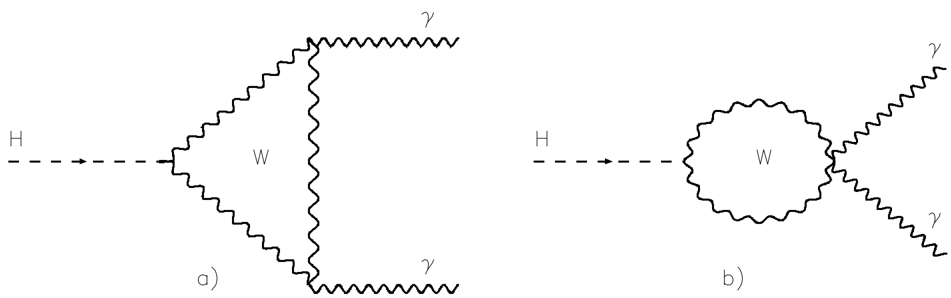


Figure 2: The triangle and bubble graphs contributing to one loop amplitude of the decay $H \to \gamma\gamma$.

In these expressions, the first rows represent the standard model vertices. The decay amplitude is then given by the formula

$$\mathcal{M}(H \to \gamma\gamma) = \varepsilon^{\star\mu_1}(k_1)\varepsilon^{\star\mu_2}(k_2)\mathcal{M}_{\mu_1\mu_2}, \tag{49}$$

where $k_{1,2}$ are the momenta of the out-going photons and $\varepsilon$'s are their polarization vectors. The polarization tensor $\mathcal{M}_{\mu_1\mu_2}$ can be splitted into two parts

$$\mathcal{M}_{\mu_1\mu_2} = \mathcal{M}^a_{\mu_1\mu_2} + \mathcal{M}^b_{\mu_1\mu_2}, \tag{50}$$

where $\mathcal{M}^a_{\mu_1\mu_2}$ is the contribution of the triangle shown in Fig. 2a (and correspondig crossed graph) and $\mathcal{M}^b_{\mu_1\mu_2}$ corresponds to the bubble in Fig. 2b, i.e.

$$\begin{aligned}\mathcal{M}^a_{\mu_1\mu_2} &= \int \frac{\mathrm{d}^{\mathrm{D}}l}{(4\pi)^{\mathrm{D}}} \frac{\mathcal{A}^a_{\mu_1\mu_2}(k_1,k_2,l)}{\left[(l+k_1)^2 - m_W^2\right]\left[(l-k_2)^2 - m_W^2\right]\left[l^2 - m_W^2\right]} \\ &\quad + \int \frac{\mathrm{d}^{\mathrm{D}}l}{(4\pi)^{\mathrm{D}}} \frac{\mathcal{A}^a_{\mu_2\mu_1}(k_2,k_1,l)}{\left[(l+k_2)^2 - m_W^2\right]\left[(l-k_1)^2 - m_W^2\right]\left[l^2 - m_W^2\right]} \end{aligned} \tag{51}$$

$$\mathcal{M}^b_{\mu_1\mu_2} = \int \frac{\mathrm{d}^{\mathrm{D}}l}{(4\pi)^{\mathrm{D}}} \frac{\mathcal{A}^b_{\mu_1\mu_2}(k_1,k_2,l)}{\left[l^2 - m_W^2\right]\left[(l-k_1-k_2)^2 - m_W^2\right]}. \tag{52}$$

Here

$$\begin{aligned}\mathcal{A}^a_{\mu_1\mu_2}(k_1,k_2,l) &= \mathrm{i}gm_W \left(g_{\alpha\mu} - \frac{(k_1+l)_\alpha(k_1+l)_\mu}{m_W^2}\right)\Gamma^{(3)}_{\mu\nu\mu_1}(k_1+l,-l,-k_1)\left(g_{\upsilon\kappa} - \frac{l_\nu l_\kappa}{m_W^2}\right) \\ &\quad \times\Gamma^{(3)}_{\kappa\lambda\mu_2}(l,-l+k_2,-k_2)\left(g_{\lambda\alpha} - \frac{(l-k_2)_\lambda(l-k_2)_\alpha}{m_W^2}\right)\end{aligned} \tag{53}$$

and

$$\begin{aligned}\mathcal{A}^b_{\mu_1\mu_2}(k_1,k_2,l) &= \mathrm{i}gm_W \left(g_{\alpha\sigma} - \frac{(k_1+k_2+l)_\alpha(k_1+k_2+l)_\sigma}{m_W^2}\right) \\ &\quad \times\Gamma^{(4)}_{\sigma\beta\mu_1\mu_2}(k_1+k_2+l,-l,-k_1,-k_2)\left(g_{\beta\alpha} - \frac{l_\beta l_\alpha}{m_W^2}\right).\end{aligned} \tag{54}$$

Using the standard Feynman parametrization and shifting the loop momenta we get the following reprezentation, which allows for symmetric integration over the loop momentum according to the standard formulas for the $D$-dimensional integration:

$$\mathcal{M}^a_{\mu_1\mu_2} = 2\int_0^1 \mathrm{d}u u \mathrm{d}v \int \frac{\mathrm{d}^{\mathrm{D}}l}{(4\pi)^{\mathrm{D}}} \frac{\mathcal{A}^a_{\mu_1\mu_2}(k_1,k_2,l-(uvk_1-u(1-v)k_2)) + ((k_1,\mu_1)\leftrightarrow(k_2,\mu_2))}{(l^2-C^a(u,v))^3}, \tag{55}$$

where

$$C^a(u,v) = m_W^2 - m_H^2 u^2 v(1-v) \tag{56}$$

and

$$\mathcal{M}^b_{\mu_1\mu_2} = \int_0^1 \mathrm{d}u \int \frac{\mathrm{d}^{\mathrm{D}}l}{(4\pi)^{\mathrm{D}}} \frac{\mathcal{A}^b_{\mu_1\mu_2}(k_1,k_2,l-u(k_1+k_2))}{(l^2-C^b(u))^2}, \tag{57}$$

where

$$C^b(u) = m_W^2 + m_H^2 u(1-u). \tag{58}$$

The rest of the calculation ( *i.e.* expansion of the numerators of the integrands (55) and (57), the symmetric integration over the loop momenta, the integration over the Feynman parameters and extraction of the finite and divergent parts) was performed using *Mathematica*. We also used the *Mathematica* package *FeynCalc* [21], which proves to be extremely useful for this purpose.

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